An Assignment of Probabilities to Alternative Assumptions concerning the Contribution of Sardine Spawning Biomass on the South Coast to Recruitment on the West Coast

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Summary

The framework put forward in FISHERIES/2016/OCT/SWG-PEL/47rev (MARAM/IWS/DEC16/Sardine/P6) is applied to spawning stock biomass and recruitment outputs from a current sardine assessment which does not fit a stock-recruitment curve internally. Four different stock-recruitment curves are fitted to those outputs to inform on the relative probabilities of different values for p, the proportion of South Coast spawning biomass contributing to West Coast recruitment. Aside from the Hockey-stick model which provides no discrimination, the models all favour values of p closer to 1, though do not exclude the possibility that p=0. If assessment outputs for the years 1983-85, which are of lesser reliability, are excluded from consideration, higher values of p become favoured to a greater extent, and there is notably less indication of a sharp drop in recruitment at the lowest biomasses.

Introduction

FISHERIES/2016/OCT/SWG-PEL/47rev (MARAM/IWS/DEC16/Sardine/P6) set out a draft framework for assigning probabilities (plausibility weightings) to alternative assumptions concerning the contribution of sardine spawning biomass on the South Coast to recruitment on the West Coast. In particular it used the results from an example assessment to show how fitting stock-recruitment curves could inform on these probabilities. This document extends that work in three ways.

a) The fits are now to spawning stock biomass and recruitment outputs from a current assessment which does not fit a stock-recruitment curve internally (FISHERIES/2016/NOV/SWG-PEL/57).

b) The fits to stock-recruitment relationships now include ones to a generalized Beverton-Holt curve with a shape parameter that can allow, inter-alia, for domed Ricker-like shapes, as well as ones to the Hockey-stick, Beverton-Holt and Ricker relationships considered previously.

c) A variant is considered which fits to the assessment outputs excluding those for the first three years (1983-85).

Methods

Details of methods are essentially unchanged, and are set out in the Appendix. This now includes specification of the details of the further Generalised (Beverton-Holt) relationship now considered.

It is evident from the plot (Figure A.1) of spawning stock and recruitment outputs for the West Coast that the values for the first three years, particularly those for 1983 for which the recruitment estimate is very low, are influential on the curves' shapes because they impact substantially on this shape close to the origin. However the nature of the assessment is such that the values for these years are poorly determined because of limited information content in the data concerning the starting numbers-at-age vector; hence the inclusion here of the results for a variant that ignores the values for these first three years in the fitting process.

Results

Results from the fits of the four stock-recruitment models considered are reported in Table A.2 and illustrated in Figure A.2. Of note is the high value of the α parameter for the Generalised model; this goes to its upper bound when the variant without data for 1983-85 is considered.

Likelihood profiles as a function of the parameter p, the proportion of South Coast spawning biomass that contributes to West Coast recruitment, are shown in Figure A.3. Figure A.4 shows Akaike-weighted probabilities for different values of p based on the Generalised model, both for a uniform prior and for a (truncated) normal prior intended to illustrate that which might be developed from further consideration of the hydrodynamics model for egg transport.

Discussion

It is evident from Figure A.3 that, aside from the Hockey-stick model which is unable to discriminate between different values for p, the model fits favour larger values for p, and the more so when the 1983-85 data are omitted. For the Ricker model in the latter case, values of p below about 0.1 fall outside the 95% CI for the best estimate at p=1.

The deliberate shape generality of the Generalised model renders this the best candidate for providing plausibility weightings for different values of p. These are shown in Figure A.4, and favour higher values of p relatively strongly, though do not exclude the possibility of low values including p=0. However once combined with the illustrative prior used to reflect possible information from the hydrodynamics model for egg transport, the bulk of the plausibility weighting shifts towards lower values of p.

The plausibility weightings are slightly higher for low values of p when the values for 1983-85 are omitted. Of interest in these last circumstances is that for the Generalised model especially, there is notably less indication of a sharp drop in recruitment at the lowest biomasses.

Acknowledgment

Andrea Ross-Gillespie assisted with the computations and development of the plots.

Appendix: Methodology

Fitting the recruitment curves

Estimates for recruitment and spawning biomass are available for the West and South Coast for the years 1983-2015 (Table A.1 and Figure A.1). Four curves are fitted to this spawning biomass and West Coast recruitment information:

1. Hockey stick:

$$R_y^{mod} = \begin{cases} \alpha S_y & \text{for } S_y \leq \beta/\alpha \\ \beta & \text{for } S_y > \beta/\alpha \end{cases} \tag{1}$$

2. Beverton-Holt

$$R_y^{mod} = \frac{\alpha S_y}{1 + \frac{\alpha}{\beta} S_y} \tag{2}$$

3. Ricker

$$R_y^{mod} = \alpha S_y e^{-\beta S_y} \tag{3}$$

4. Generalised

$$R_y^{mod} = \frac{\alpha S_y}{1 + \frac{\alpha}{\beta} (S_y)^\gamma} \tag{4}$$

where α and β and γ are estimable parameters and S_y is a measure of the spawning biomass contributing to the West Coast recruitment given by:

$$S_y = S_y^{west} + pS_y^{south} \tag{5}$$

where S_y^{west} and S_y^{south} are the respective West and South Coast spawning biomass estimates, and p is an estimable parameter that can take on values between 0 and 1.

Assuming that residuals are log-normally distributed, the negative log-likelihood is given by:

$$-lnL = \sum_y \left(ln\sigma_y + \left(lnR_y^{west} - lnR_y^{mod} \right)^2 / (2\sigma_y^2) \right) \tag{6}$$

where R_y^{west} is the recruitment estimate for the West Coast in year y and σ_y is given by:

$$\sigma_y = \begin{cases} \sigma_2 & \text{for } 2000 \leq y \leq 2004 \\ \sigma_1 & \text{otherwise} \end{cases} \tag{7}$$

(It has been conventional to assume a higher variance for these five "peak-related" years in previous analyses.)

Closed form equations are available for the maximum likelihood estimates for the σ_1 and σ_2 parameters:

$$\sigma_i = \sqrt{\frac{1}{n} \sum_{y \in Y_i} \left(lnR_y^{west} - lnR_y^{mod} \right)^2} \tag{8}$$

where the set $Y_2 = \{2000; 2004\}$ and Y_1 contains the remaining years.

Results from these fits are given in Table A.2 and Figures A.2 and A.3.

Probability histogram for p (Akaike weights)

The following steps may be taken to obtain a probability histogram for p, weighted across (as an example) three of the different stock-recruitment models:

1. Assume that the three models ($\text{model}_i \in$ {Hockey stick, Beverton-Holt, Ricker}) and 11 p values ($p_j \in$ {0.0, 0.1; ..., 1.0}) are equally likely each with a consequent prior weight of $W_{ij}^{prior} = 1/33$ (i.e. a uniform prior distribution[1]).

2. Multiply this prior weight by the Akaike weight for each (model_i, p_j) combination:

$$W_{ij} = W_{ij}^{prior} e^{-\Delta lnL_{ij}} \tag{9}$$

   where ΔlnL_{ij} is the difference between the negative log likelihood for (model_i, p_j) and the lowest negative log likelihood value for the 33 (model_i, p_j) combinations.

3. Normalise the resulting 33 W_{ij} values to sum to 1.

4. For each p_j sum over the models_i to calculate $W_j = \sum_i W_{ij}$, where W_j provides the value for the probability histogram at p_j.

These steps were applied to just the Generalised model (i.e. i above is 1), with 21 values of p on the interval [0.1]. Results of these weightings are shown in Figure A.4.

[1] This document also reports on the probability histogram when a normal prior distribution with mean 0.083 and standard deviation 0.3 is assumed. In this case, the prior weight of (1/33) is replaced by $W_{ij}^{prior} = e^{(p_j-0.2)^2/(2(0.3)^2)}/(\sqrt{2\pi(0.3)^2})$, normalised so that $\sum_{ij} W_{ij}^{prior} = 1$.

Table A.1: Estimates of sardine recruitment (in billions) and spawning biomass (in thousand tons) available for the analysis from an assessment which did not estimate any stock-recruitment relationship internally (from FISHERIES/2016/NOV/SWG-PEL/57). Note that the year stated corresponds to the November of the year in which the survey took place, and to which the associated resultant recruitment is also assigned.

Year	Recruitment		Spawning biomass		Year	Recruitment		Spawning biomass	
	West	South	West	South	..cont.	West	South	West	South
1983	1.63	0.00	1.46	0.01	1999	64.41	25.00	179.76	254.89
1984	8.15	0.00	25.14	0.22	2000	173.91	0.00	180.27	298.84
1985	13.53	0.00	9.43	10.37	2001	125.97	7.76	107.00	359.92
1986	14.83	0.13	30.67	17.91	2002	117.93	2.76	173.86	666.57
1987	5.37	0.49	103.46	14.79	2003	15.63	3.45	192.70	702.48
1988	9.58	2.85	83.73	5.13	2004	10.21	6.64	121.24	1168.91
1989	6.29	1.52	131.02	15.09	2005	18.53	15.24	42.69	637.50
1990	8.88	3.54	108.81	18.24	2006	9.25	8.74	34.75	254.35
1991	21.77	2.15	62.60	21.77	2007	22.01	1.80	26.12	149.77
1992	38.94	4.46	58.10	34.68	2008	17.98	11.62	21.71	109.64
1993	7.65	2.29	143.00	38.54	2009	64.09	1.32	83.65	197.74
1994	41.82	1.45	228.22	84.50	2010	7.79	4.39	170.11	103.80
1995	16.07	4.33	163.45	61.94	2011	27.09	3.47	424.92	184.01
1996	73.51	0.00	178.42	52.61	2012	18.09	0.19	256.82	73.24
1997	53.37	1.75	234.04	70.96	2013	8.26	13.30	100.39	144.25
1998	43.42	9.13	246.04	88.48	2014	20.78	16.82	96.48	221.35

Table A.2: Parameter estimates for the four different forms of stock-recruitment relationship and three different values for p for each form.

Input data	Form	p	α	β	γ	σ_1	σ_2	-lnL
Include all years	(a) Hockey-stick	0.0	1.269	19.395	-	0.756	1.551	10.65
		0.5	1.018	19.394	-	0.756	1.551	10.64
		1.0	0.626	20.105	-	0.758	1.528	10.63
	(b) Beverton-Holt	0.0	1.357	24.390	-	0.744	1.479	9.98
		0.5	0.926	23.993	-	0.732	1.469	9.48
		1.0	0.807	23.731	-	0.727	1.463	9.30
	(c) Ricker	0.0	0.479	0.006	-	0.854	1.339	13.20
		0.5	0.282	0.004	-	0.832	1.295	12.33
		1.0	0.213	0.002	-	0.843	1.244	12.47
	(d) Generalised	0.0	1.975	9.354	0.811	0.739	1.462	9.72
		0.5	2.542	3.465	0.646	0.710	1.415	8.50
		1.0	2.371	3.321	0.656	0.703	1.401	8.16
Exclude first three years (1983-1985)	(a) Hockey-stick	0.0	0.828	20.273	-	0.783	1.523	10.74
		0.5	1.000	20.173	-	0.783	1.526	10.75
		1.0	1.000	20.173	-	0.783	1.526	10.75
	(b) Beverton-Holt	0.0	1.298	24.980	-	0.776	1.469	10.32
		0.5	0.274	38.404	-	0.754	1.386	9.36
		1.0	0.212	38.886	-	0.744	1.362	8.95
	(c) Ricker	0.0	0.385	0.005	-	0.843	1.363	11.94
		0.5	0.193	0.002	-	0.766	1.312	9.46
		1.0	0.151	0.002	-	0.752	1.249	8.78
	(d) Generalised	0.0	10.000*	6.542	0.753	0.764	1.463	9.93
		0.5	10.000*	2.023	0.551	0.739	1.413	8.98
		1.0	10.000*	1.938	0.565	0.734	1.402	8.77

*Constraint boundary

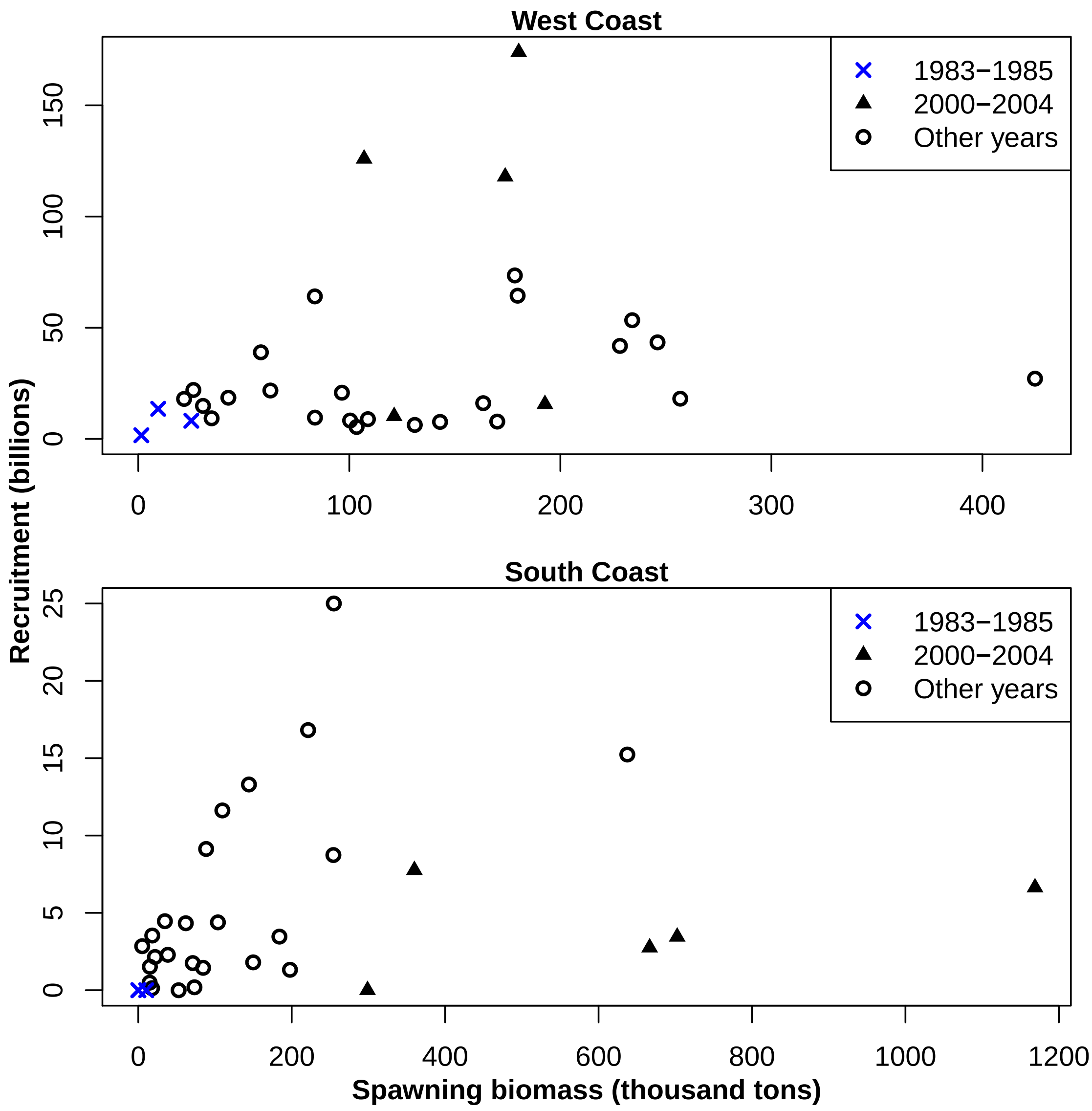

Figure A.1: Plots of recruitment vs spawning biomass for the assessment without an internally fitted stock-recruitment relationship. Blue crosses indicate point corresponding to the years 1983-1985. Filled black triangles indicate the years 2000-2004, which have conventionally been taken to exhibit a higher variance about the stock-recruitment relationship.

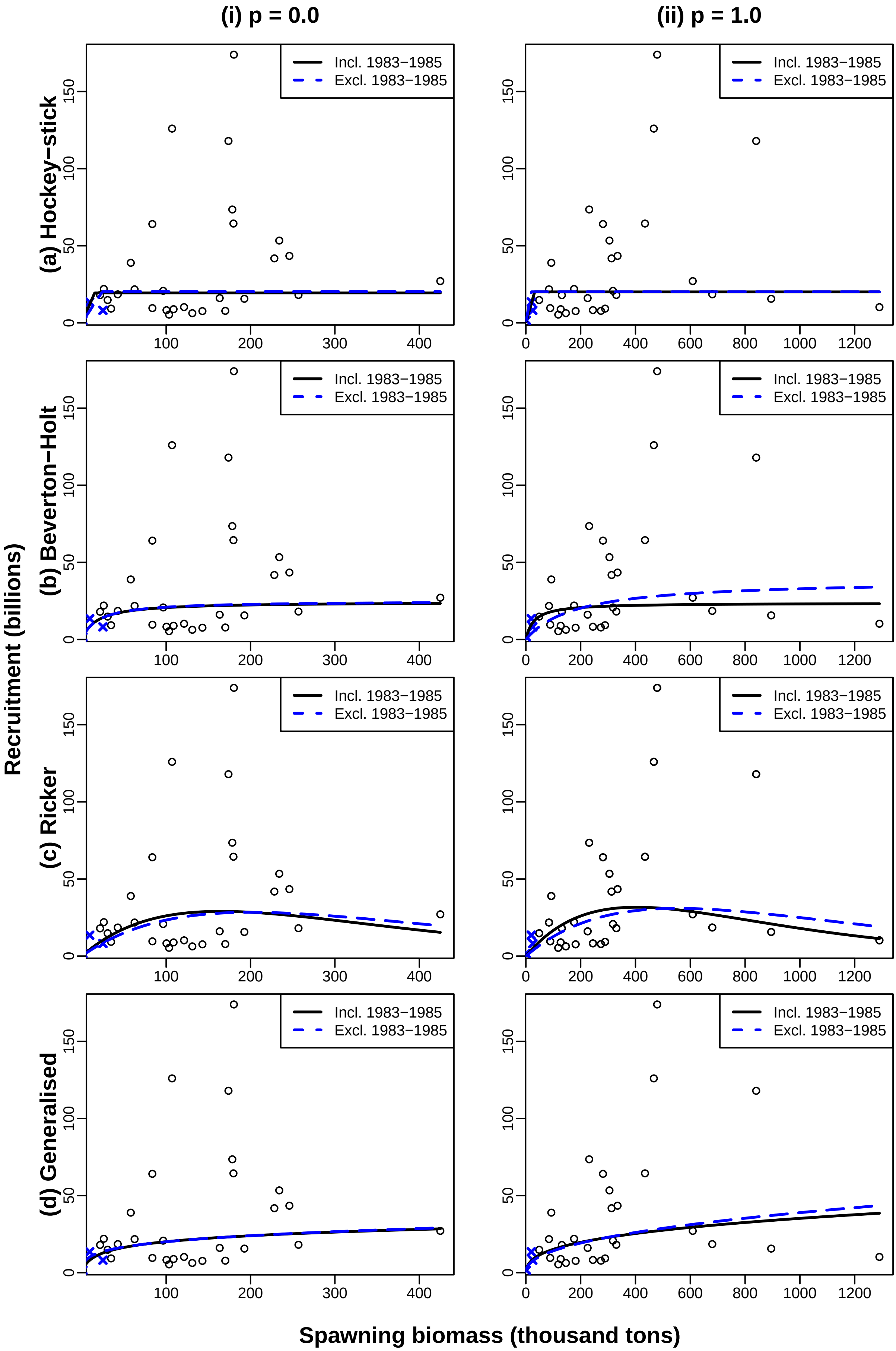

Figure A.2: Plots of the fits of the estimated stock-recruitment relationships to the spawning biomass and recruitment estimates from the assessment without an internally fitted stock-recruitment relationship. Fits are shown for the case where the 1983-1985 points are included (solid black lines) and where they have been excluded (dashed blue lines) and for two values of p ($p = 0$ and $p = 1.0$).

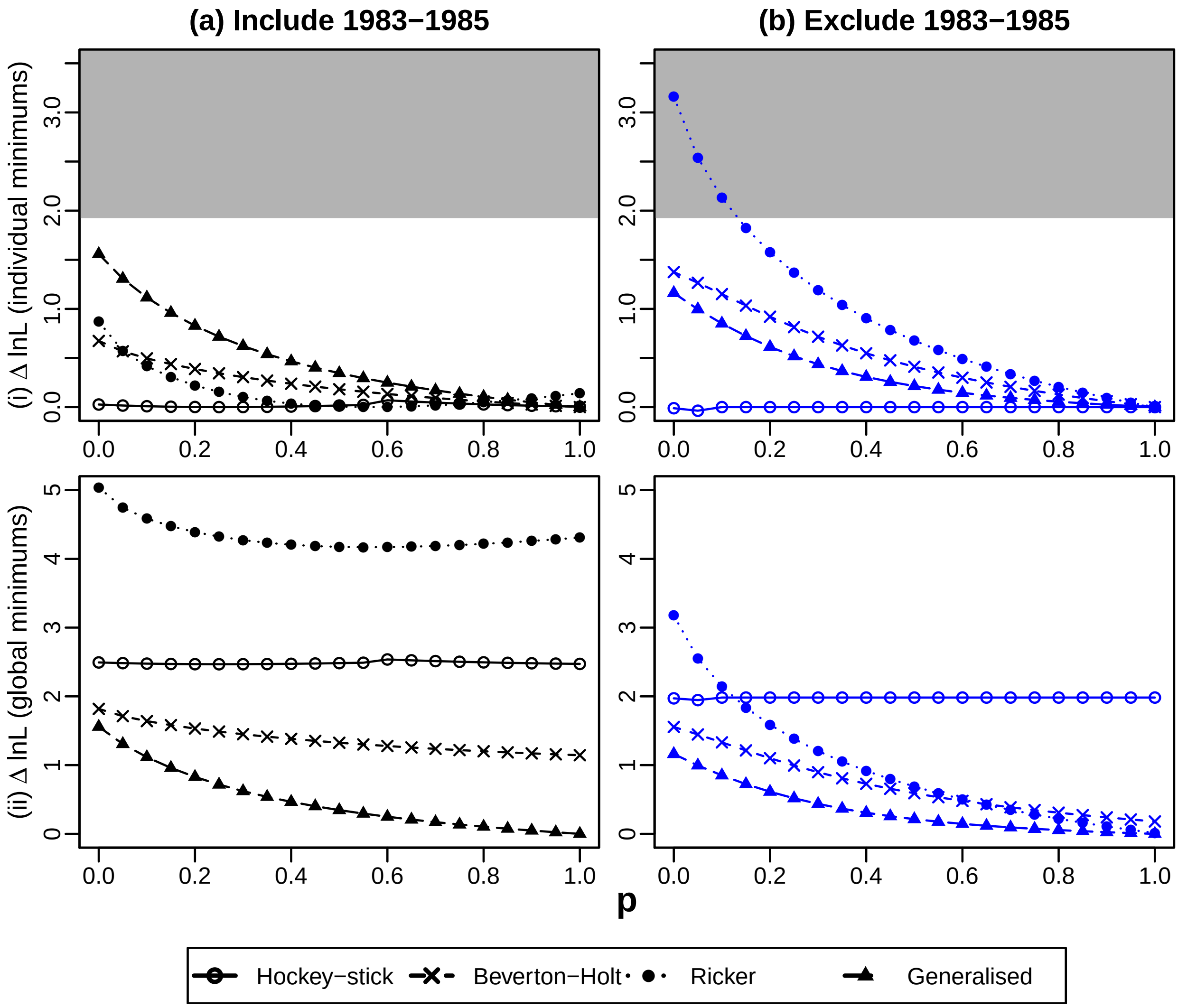

Figure A.3: Negative log-likelihood profiles for the p parameter for the four different stock-recruitment relationships. The top two plots show the negative log-likelihoods for each stock-recruitment relationship relative to the lowest value over the range of [0, 1] for p for that relationship. The bottom two plots show these negative log-likelihoods relative to the global minimum across all the stock-recruitment relationships. The two left-hand plots (in black) are for the case where all the data have been utilised, and the right-hand plots (in blue) for the case where the years 1983-1985 are excluded. Points that fall into the grey-shaded region in the top two plots correspond to values of p which are outside the associated 95% CI for the best estimate for p.

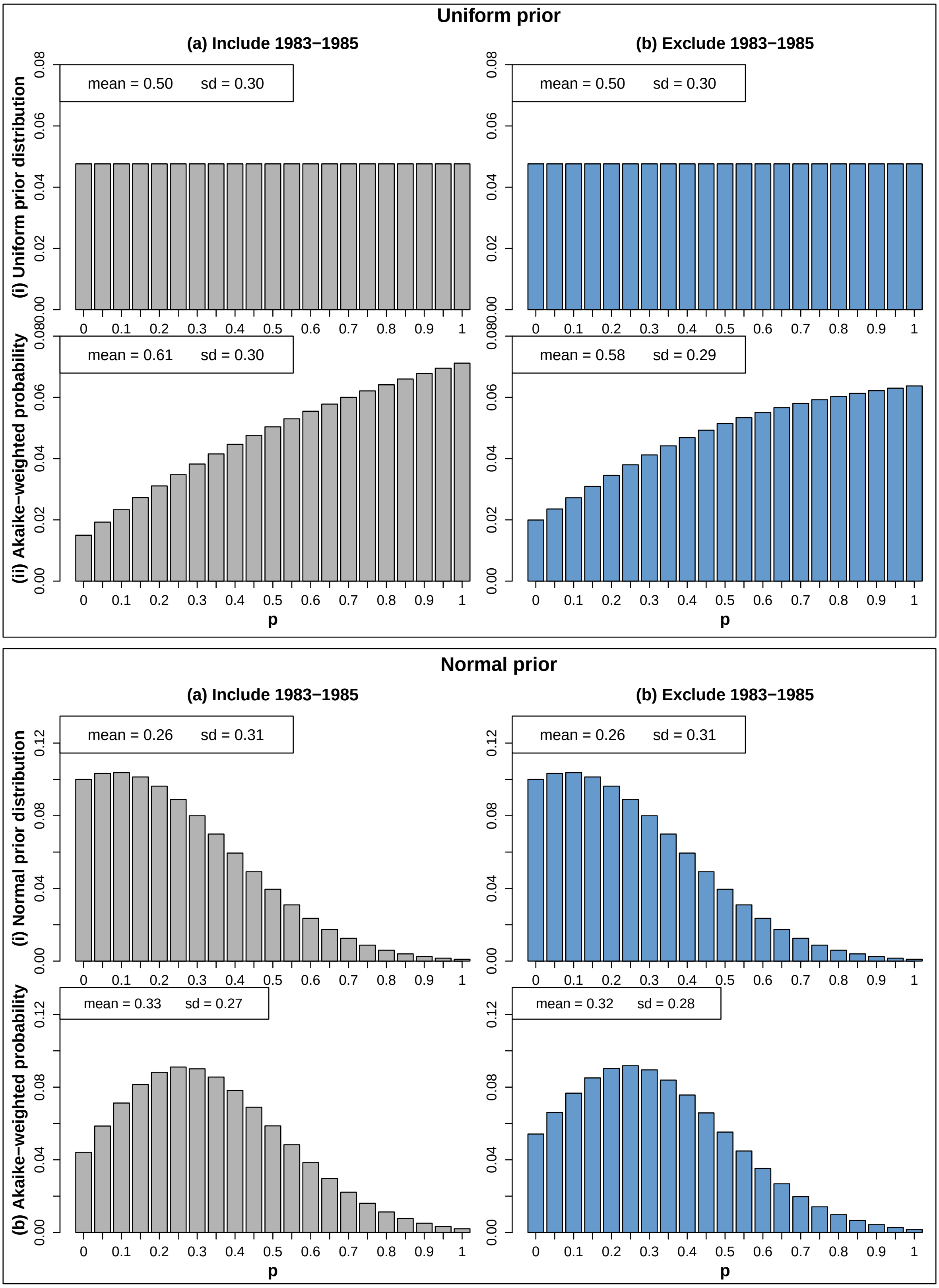

Figure A.4: Probability histogram for p calculated using Akaike weights as described in the Appendix for the **Generalised model** under the assumption of a uniform prior distribution (top panel of four plots) and under the assumption of a normal prior distribution with a mean of 0.083 and a standard deviation of 0.3 (bottom panel of four plots). In each panel, the top two plots show the uniform prior distribution and the bottom two the distribution when this prior is multiplied by the Akaike weights. The plots with the grey bars on the left are for the case where all the data have been utilised and the blue plots on the right are for the case where the 1983-1985 years have been excluded. The mean and standard deviation for each distribution are reported in the top left legend. Note that this Figure has been updated from the one shown in FISHERIES/2016/NOV/SWG-PEL/66 after it was realised that the weighting had accidentally been performed across all four models instead of just the Generalised model.