Relationship of equation (1) of PENG/P4 to equation (4) of PENG/P7

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Equation (1) of Peng/P4:

\[ y_{i,j,k,l} = \alpha + b_j + b_{j,k} + \beta_1 x_j + \beta_2 z_i + \beta_3 x_j z_i + \beta_4 \frac{S_j}{S} + \beta_5 \frac{A_j}{A} + \epsilon_{i,j,k,l} \]

Omit information not available for OM4 (month, sardine and anchovy abundance):

\[ y_{i,j,l} = \alpha + b_j + \beta_1 x_j + \beta_2 z_i + \beta_3 x_j z_i + \epsilon_{i,j,l} \]

Change to OM4 notation \( j \rightarrow y, l \rightarrow j, y \rightarrow F \)

\[ F_{i,y,j} = \alpha + b_y + \beta_1 x_y + \beta_2 z_i + \beta_3 x_y z_i + \epsilon_{i,y,j} \]

Re-arrange order

\[ F_{i,y,j} = \alpha + \beta_2 z_i + b_y + \beta_1 x_y + \beta_3 x_y z_i + \epsilon_{i,y,j} \]

Define \( a_i = \alpha + \beta_2 z_i \) and \( \delta(X_{i,y}) = \beta_1 x_y + \beta_3 x_y z_i \) (where \( \delta(X_{i,y}) \) is defined as in PENG/P7)

\[ F_{i,y,j} = a_i + b_y + \delta(X_{i,y}) + \epsilon_{i,y,j} \]

Equation (4) of PENG/P7 (corresponding to OM4) is:

\[ F_{i,y,j} = a_i + b_y + \eta_{i,y} + \delta(X_{i,y}) + \epsilon_{2i,y,j} \]

Note:

i. \( x_y, z_i \) and \( X_{i,y} \) are binary variables
ii. OM4 assumes the closure effect \( \delta \) to be the same for both islands; PENG/P4 assumes the effect to differ for the two islands (\( \beta_1 \) for the one, and \( \beta_1 + \beta_3 \) for the other).